647510 **0301207C1010** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination
July-2025 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one out of two. **(07)**

(1) Derive equation of the tangent plane at any point (α,β,γ) of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

(2) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ And passing through a guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0$; $z=0$.

Que.1(B) Answer any one out of two. **(04)**

(1) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

(2) Find equation of right circular cylinder passing through (3,4,5) with radius 2 And axis is parallel to $\{(3,4,5+k)/k\in R\}$.

Que.1(C) Answer any Three out of Five. **(03)**

(1) Find equation of sphere where extermities of a diameter is (1,1,2),(2,2,1).

(2) Find centre and radii of the sphere $x^2+y^2+z^2=9$.

(3) Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ And the point (1,2,3).

(4) Define : cylinder

(5) Write equation of right circular cylinder with radius r and axis X axis.

Que.2(A) Answer any one out of two. **(07)**

(1) State and prove Euler's theorem for homogeneous function of two variables.

(2) If f(x,y)=0 then obtain formula to find $\frac{dy}{dx}$ *and* $\frac{d^2y}{dx^2}$.

Que.2(B) Answer any one out of two. **(04)**

(1) If $f(x,y)=(x^2+y^2)tan^{-1}\frac{y}{x}$ *when* $x\neq 0$ *and* $f(0,0)=0$ then discuss The continuity of f at (0,0).

(2) If u=f(y-z, z-x, x-y) then find value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

Que.2(C) Answer any Three out of Five. **(03)**

(1) Define : partial differentiation.

(2) Define : Homogeneous function.

(3) Define : Total differential.

(4) State young's theorem for partial differentiation.

(5) Obtain $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$ *from* $f(x,y,z)=0$ by without giving simplification.

Que.3(A) Answer any one out of two. **(07)**

(1) State and prove Taylor's theorem.

(2) Find the shortest distance from origin to the surface $xyz^2=2$.

Que.3(B) Answer any one out of two. **(04)**

(1) Find the greatest and smallest value of the function f(x,y)=xy takes on Ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$.

(2) if $u=\frac{yz}{x}$, $v=\frac{zx}{y}$, $w=\frac{xy}{z}$ then find value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Que.3(C) Answer any Three out of Five. **(03)**

(1) Explain approximate value of z=f(x,y) which is differentiable at (x,y).

(2) Explain global maxima and minima.

(3) Define : Jacobian.

(4) if x=rcosθ , y=rsinθ then find value of $\frac{\partial(x,y)}{\partial(r,\theta)}$.

(5) if $f(x,y)=x^2+2y^2-x$ then find critical point of f.

Que.4(A) **Answer any one out of two.** **(07)**

(1) State and prove Distributive law for matrices.

(2) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a skew Hermitian matrix.

Que.4(B) **Answer any one out of two.** **(04)**

(1) Find Echelon form of Matrix $\begin{bmatrix} 1 & 3 & -3 & -1 & -4 \\ 1 & 4 & 1 & -2 & -2 \\ 2 & 9 & 0 & -5 & -2 \end{bmatrix}$.

(2) Find Rank of Matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

Que.4(C) **Answer any Three out of Five.** **(03)**

(1) Define : Periodic Matrix

(2) Define : idempotent Matrix

(3) Define : Symmetric Matrix

(4) Define : Rank of a Matrix

(5) Define : involutory Matrix

Que.5(A) **Answer any one out of two.** **(07)**

(1) Prove that there is unique Eigen value with respect to any Eigen vector of Given Matrix.

(2) State and prove condition of consistency for the system of equation AX=B in usual notation.

Que.5(B) **Answer any one out of two.** **(04)**

(1) Using Cayley-Hamilton theorem find inverse of Matrix $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

(2) Find Eigen values and Eigen vectors of Matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

Que.5(C) **Answer any Three out of Five.** **(03)**

(1) Define : characteristic equation.

(2) Define : Homogeneous and Non Homogeneous Linear equation.

(3) State Cayley-Hamilton Theorem.

(4) Show that the matrix A and A^T have the same Eigen values.

(5) State the condition under which a system of Non Homogeneous Linear Equation will have (i) No solution (ii) a Unique solution.

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